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**OPTICAL PHYSICS** 

# Ultrabroadband infrared near-field spectroscopy and imaging of local resonators in percolative gold films

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Percolation processes are ubiquitous in nature and are responsible for many critical phenomena such as first-order phase transitions and infectious epidemic networks. The optical properties of a percolative medium can generally be captured by the effective medium approximation (EMA) when the degree of percolation and the properties of the constituent materials are properly addressed. However, the important local collective responses of nanoclusters in the deep subwavelength regime are often only phenomenologically addressed in the standard EMA formalism. A comprehensive method that measures local light-matter interactions and registers how the local responses influence global optical properties has yet to be established on a firm basis. In this paper, we use infrared nano-imaging/spectroscopy to investigate percolative gold films in the vicinity of the critical percolation threshold. We demonstrate experimentally and theoretically that the near-field spectra yield quantitative information of the characteristic length scale of the local gold clusters and their relative oscillator strengths. As a result, EMA analysis can be augmented with near-field nano-spectroscopy to yield better predictability of the far-field reflection spectrum at the corresponding spectral range. © 2019 Optical Society of America

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## **1. INTRODUCTION**

Percolative metal-dielectric composites possess interesting electromagnetic properties. For example, the insulator-to-metal transitions in percolative metal thin films (PMFs) yield tunable dielectric constants that are anomalously high in the vicinity of the phase percolation [1]. Among complex metal-dielectric composites, random percolative metal-dielectric thin films are exceptionally difficult to study because their nonuniform cluster size, irregular shape, and inhomogeneous distributions pose formidable obstacles for rigorous ab initio treatments [2,3]. In addition, local metal clusters at a length scale significantly smaller than the wavelength can play a dominant role in determining the plasmonic responses of the PMFs [4-9], which are difficult to address using areal averaging methods. Optical probes that can offer quantitative measurements of the microscopic details are therefore crucial for better modeling of the metal-insulator mixtures.

In a simple picture, PMFs can be modeled as a random arrangement of conducting bonds on an infinite grid. As the

fraction of the metal distribution f increases from 0 to 1, an electrically connected network gradually forms, and the system undergoes a transition from insulating to conducting. This transition occurs at a critical fraction  $f_c$  whose value depends on many factors such as the system dimensionality, lattice geometry, and the correlation between sites (e.g.,  $f_c = 0.5$  for a two-dimensional square lattice without correlation). The macroscopic physical properties of these systems, such as film sheet conductivity, can be derived from effective approaches such as the classical effective medium approximation (EMA) [10]. Although qualitative macroscopic properties can usually be well understood through EMA, local characteristics involving mesoscale collective electron interactions are generally overlooked [11-13]. This is why metamaterials or photonic crystals, for example, cannot be described satisfactorily using conventional EMA formulations. On the contrary, the metamaterial resonances can be described well in terms of local oscillators using equivalent inductor and capacitor (LC) circuit



**Fig. 1.** (a)–(c) SEM images of three representative samples across the percolation threshold; (d) table for sample sheet resistance and mean thickness.

analysis or antenna theory at the single-unit-cell level (usually the inter-cell interaction is neglected) [14]. In the case of random PMFs, however, no unit cell can be specified, and a rigorous microscopic description is mathematically expensive. One way to solve this dilemma is to use a phenomenological and less intuitive EMA that describes the local variations of the inclusion [1,15]. Here we demonstrate a new method to understand the local and global properties of PMFs: starting from the effective dielectric function using EMA, by incorporating the near-field-resolved local resonances into our analysis, the IR reflectance can be faithfully reproduced.

A series of gold PMFs on c-cut sapphire substrates are prepared by the thermal vacuum deposition method. Their sheet resistance is closely monitored during the deposition process so that they span across the percolative threshold. This is verified by scanning electron microscope (SEM) and DC resistance measurements, where samples with similar morphology show dramatically different sheet resistances [Figs. 1(a)–1(c)]. Films exhibiting infinite and finite sheet resistance are classified as insulating films (IFs) and conductive films (CFs), respectively. Here we focus our attention on the optical properties of 4 (out of 9) characteristic samples with sheet resistance 20  $\Omega/\Box$  (sample A), 14.5 k $\Omega/\Box$ (sample B), and  $\infty \Omega/\Box$  (samples C and D). The detailed growth conditions and DC measurements of the full sample set will be reported in a separate paper.

## 2. FAR-FIELD INFRARED SPECTROSCOPY

We first carry out conventional Fourier-transform infrared spectroscopy (FTIR) and apply EMA to interpret the far-field reflectance R. The schematic of the model is depicted in Fig. 2(a). The IR far-field reflectance spectra of the set of gold films are shown in Fig. 2(b). With increasing percolation levels, R is greatly enhanced at higher frequencies (> 900 cm<sup>-1</sup>) (indicated by the red arrow). The spectrum of the c-cut sapphire

substrate is also included as a reference (gray curve). It is clear that the IR active phonon response of the bare sapphire substrate shows a high reflectance between 750 and 900 cm<sup>-1</sup> and cutoffs at  $\sim 1000$  cm<sup>-1</sup> (Reststrahlen band). The spectra of the thin gold films on sapphire, on the other hand, do not show flat optical responses as from uniform metal films but instead show spectral features reminiscent of sapphire phonons (pink to red curves). There is a pronounced frequency dip at  $\sim 800$  cm<sup>-1</sup> that red-shifts with increasing levels of percolation and diminishes for the most conducting film (Sample A). We attribute the red-shift to the interplay between the sapphire phonon mode and the gold plasmonic response.

To understand the far-field IR spectra of the percolating gold films, a simple two-dimensional Bruggeman EMA is employed. We first approximate the gold layer as an effective medium that is a mixture of gold and air. According to Bruggeman's formulation with circular inclusions, we have

$$\frac{f(\varepsilon_g - \varepsilon_{\rm eff})}{\varepsilon_g + \varepsilon_{\rm eff}} + \frac{(1 - f)(\varepsilon_{\rm air} - \varepsilon_{\rm eff})}{\varepsilon_{\rm air} + \varepsilon_{\rm eff}} = 0,$$
 (1)

where f is the filling fraction of gold [12,16,17]. In this model, the effective percolation threshold is  $f_c = 0.5$ , which is lower than the expected value  $f_c^{exp} > 0.9$  estimated from the SEM images of our films. Nonetheless, this approach provides a reasonable qualitative description and a more intuitive picture than other complex models. We note here that since the gold clusters have a uniform and isotropic distribution, for simplicity, we use an EMA with isotropic circular inclusions and avoid other geometries such as elliptical and needle-like shapes. This also "turns off" the inter-cluster interactions [1].

As shown in Fig. 2(c), the far-field reflectance is calculated using the multilayer Fresnel equation considering a layer with EMA permittivity  $\varepsilon_{\text{eff}}$  on top of a sapphire substrate. Gold is described by a conventional Drude model, and the complex dielectric permittivity is given by



**Fig. 2.** (a) Schematics of percolating gold film (left) and the corresponding EMA modeling (right). (b) Far-field FTIR reflectance of a bare sapphire substrate (gray curve) and gold films on sapphire substrate (red curves). (c) Simulation of far-field reflectance in (b) using EMA with f taken between 0.49 (below percolation) and 0.51 (above percolation). (d) Calculation of IR reflectance as a function of frequency and filling fraction f. (e) A zoom-in of (d) close to  $f_c = 0.5$ . (f) Experimental data in (b) replotted as a function of frequency and sheet resistance, in comparison with (e).

$$\varepsilon_g = 1 - \frac{\omega_p^2}{\omega^2 + i\gamma\omega},$$
 (2)

where  $\gamma = \frac{1}{\tau}$  is the relaxation rate and  $\omega_p$  is the plasma frequency [18]. Over the years, inconsistent values of both  $\omega_p$  and  $\gamma$  have been experimentally measured mainly due to sample growth and experimental conditions [19,20]. Here we adopt the values from the relatively new study by Olmon *et al.* [21], where  $\tau$  is found to be ~ 14 fs and  $\omega_p = 8.45$  eV. On the other hand, sapphire exhibits anisotropy where the dielectric functions for ordinary (o) and extraordinary (e) axes are different. The dielectric function is given by

$$\varepsilon_{\mu} = \varepsilon_{\infty,\mu} \prod_{j} \frac{\omega_{jLO,\mu}^{2} - \omega^{2} - i\gamma_{jLO,\mu}\omega}{\omega_{jTO,\mu}^{2} - \omega^{2} - i\gamma_{jTO,\mu}\omega}, \qquad (3)$$

where  $\mu = o$  ( $\mu = e$ ) for o (e) axes;  $\omega_{jLO,\mu}$  ( $\omega_{jTO,\mu}$ ) and  $\gamma_{jLO,\mu}$  ( $\gamma_{jTO,\mu}$ ) are the resonant frequency and relaxation rate of the *j*th longitudinal (transverse) optical mode. Numeric values of those parameters can be found in the literature [22,23]. The reflectance is therefore calculated as

$$R = |r|^{2} = \left| \frac{r^{01} + r^{12} e^{2i\theta}}{1 + r^{01} r^{12} e^{2i\theta}} \right|^{2},$$
 (4)

where  $r^{ij} = \frac{n_i - n_j}{n_i + n_j}$  stands for the reflection coefficient between the *i*th and *j*th layer, and  $\theta = \frac{2\pi n_1 d}{\lambda}$  with *d* the film thickness and  $\lambda$  the free-space wavelength. Realistically, the mean film thickness varies from 24 to 37.5 nm. As the thickness does not drastically change the calculation result, for simplicity, here we set d = 30 nm without loss of generality.

As f increases across the percolation threshold, the spectrum changes dramatically and agrees qualitatively with the experimental data [Fig. 1(b)]. In particular, the red-shift of the dip of the resonance is reproduced well by the current model, reaching  $\sim 800 \text{ cm}^{-1}$  at  $f_c = 0.5$ . This red-shift in the far-field responses is therefore likely to be a manifestation of the percolation behavior. To better illustrate this, we plot the calculated IR reflectance as a function of frequency and f in Fig. 2(d) and zoom into the red-shifted region around  $f_c = 0.5$  in Fig. 2(e). As shown in Fig. 2(f), experimental data replotted as a function of frequency and sheet resistance shows the same trend as in Fig. 2(e). Therefore, the general far-field IR responses in our gold films are qualitatively characterized by EMA with f close to  $f_c = 0.5$ . However, the long spectral tails calculated by EMA above the  $\sim 900 \text{ cm}^{-1}$  range [Fig. 1(c)] significantly underestimate the IR reflectance from experiments [Fig. 1(b)]. As we will show below, the local plasmonic behavior of the gold clusters emerges as a dominant factor at frequencies above 900 cm<sup>-1</sup> and contributes to the spectral weight lost in EMA. These local optical responses are expected to demonstrate strong variations depending on the local geometries of the granular domains and are totally averaged out in the far-field measurements [24-27].

# 3. NEAR-FIELD INFRARED SPECTROSCOPY AND NANO-IMAGING

Near-field nano-imaging and nano-spectroscopy are therefore useful in probing the local optical responses of the gold clusters



**Fig. 3.** (a) Schematic illustration of scattering-type scanning near-field optical microscopy (s-SNOM) experiments using broadband or monochromatic light sources. (b) and (c) THz s-SNOM images of samples A and D. (d)–(f) IR s-SNOM images of samples A, B, and D. (g)–(i) AFM topographic images of samples A, B, and D.

and revealing the characteristics of their plasmonic behaviors [28-30]. The near-field experiment is schematically shown in Fig. 3(a). In the terahertz (THz) region ( $\lambda \approx 300 \ \mu m$ ), nano-imaging reveals a rather homogeneous response in all samples. In Figs. 3(b) and 3(c), we show two representative THz scattering-type scanning near-field optical microscopy (s-SNOM) images of samples A (20  $\Omega/\Box$ ) and D ( $\infty$ ). It is worth noting that although optical inhomogeneity is absent, the global THz near-field signal levels differ considerably between the two films: the one with low sheet resistance yields a nearly threefold increase in the THz signal, indicating the drastically different low-frequency film conductivity. In the mid-IR region ( $\lambda = 11 \mu m$ ), on the other hand, strong optical inhomogeneities are discovered in all films, similar to previous reports using short wavelength excitation [9,25,31–33]. Three representative samples are shown in Figs. 3(d)-3(f), in which local IR responses vary dramatically over a length scale of a few micrometers. The atomic force microscope (AFM) images acquired in the same set of samples [Figs. 3(g)-3(i)] do not show topographical variations on the same micrometer scale of the near-field "hot spots."

To understand the optical inhomogeneities in the monochromatic near-field images, we performed broadband nano-spectroscopy based on a similar setup [34]. Representative nano-IR spectra were taken at random locations within a 10  $\mu$ m × 10  $\mu$ m scan area on samples A [Fig. 4(a)], B [Fig. 4(b)], C [Fig. 4(c)], and D [Fig. 4(d)]. All the near-field spectra are normalized to that of a thick gold film to yield a physically meaningful relative amplitude. From Figs. 4(a)-4(d), three major observations are evident: (1) the sapphire phonon response is still visible but reduced in strength; (2) nano-IR spectra vary from one location to another, with the variation being more dramatic in the CFs than the IFs; (3) multiple resonant peaks at high frequencies (900 – 1400 cm<sup>-1</sup>) can be identified in the CFs [marked by arrows in Fig. 4(a)], but they are less prominent in the IFs. Those spectral peaks are absent in the far-field spectra in Fig. 2(b) and are believed to originate from the local plasmonic resonators formed by gold clusters.

## 4. THEORETICAL MODELING

We propose that the s-SNOM probes the local permittivity of the gold films, which can be described by the sum of the macroscopic effective medium background and local Lorentzian resonators as follows:

$$\varepsilon_g^n = \varepsilon_{\text{eff}} + \sum_j \frac{s_j}{\omega_{oj}^2 - \omega^2 + i\omega\gamma_j},$$
(5)

where  $s_j$  is the oscillator strength,  $\omega_{0j}$  is the resonant frequency, and  $\gamma_j$  is the damping coefficient.  $\varepsilon_{\text{eff}}$  is given by Eq. (1). We used f = 0.5 in the following calculation. Higher (lower) fcorresponds to more metallic (insulating) behavior, which only affects the overall signal intensity in the near-field measurement but has minimal influence on the resonance. This phenomenon



**Fig. 4.** (a)–(d) Near-field spectra taken at random locations in two CFs (samples A and B) and two IFs (samples C and D). All curves are shifted for clarity. Arrows in (a), (b), and (c) mark the local resonance peaks. (e) IR near-field spectra taken from a CF and bare sapphire compared with theoretically calculated spectra. (f) Far-field reflectance using only EMA (blue curve) and EMA with local resonators (red curve). For comparison, one experimental spectrum from Fig. 1(b) is also shown (red dashed curve).

has been observed similarly in an artificial percolation system with scanning microwave impedance microscopy [35]. These resonators emerge because of the strong plasmonic resonances. The near-field response can be calculated analytically or numerically by modeling the AFM tip as a point dipole, elongated spheroid, or cone [23,36–41]. Here we adopt the finite-dipole model as detailed in the work by Cvitkovic *et al.* [36]. The essence is that the scattered field, which is experimentally detected in s-SNOM, is given by

$$E_{\rm sca} \propto \left(1 + r_p\right)^2 \alpha_{\rm eff},$$
 (6)

where  $\alpha_{\text{eff}}\beta$  is the tip's effective polarizability that is proportional to the induced dipole moment on the tip.  $\alpha_{\text{eff}}$  reflects how strong the near-field tip–sample interaction is. In the dipole scattering framework, the scattered field is proportional to the dipole moment and thus proportional to  $\alpha_{\text{eff}}$ .  $r_p$  is the *p*-polarized reflection coefficient, and  $(1 + r_p)^2$  is commonly referred to as the "far-field factor" due to direct surface reflection. This factor is found to play a minor role and so is ignored for the sake of simplicity [42].  $\beta$  is the sample response function, which is related to sample permittivity. For bulk samples with permittivity  $\varepsilon$ ,  $\beta = (\varepsilon - 1)/(\varepsilon + 1)$  is the quasi-static reflection coefficient. For our gold thin film/sapphire structure,  $\varepsilon$  as an approximation can be interpreted as an effective value derived directly from the Fresnel equation

$$\varepsilon^* = \varepsilon_g^n \frac{\varepsilon_s + \varepsilon_g^n \tanh(q^*d)}{\varepsilon_g^n + \varepsilon_s \tanh(q^*d)},$$
(7)

where  $\varepsilon_s = \sqrt{\varepsilon_o \varepsilon_e}$  is the averaged permittivity of sapphire [42], *d* is the film thickness, and  $q^* \sim 1/(10a)$  is the phenomenological effective in-plane momentum [43]. For a direct comparison with the experimental near-field spectrum,  $\alpha_{\text{eff}}$  needs to be demodulated to the second harmonics of the tip tapping frequency and normalized to a gold reference. As a demonstration, in Fig. 4(e) we compare our model calculation (solid curves)

$$S_2 \propto (\alpha_{\rm eff})_{\rm 2nd} / (\alpha_{\rm reference})_{\rm 2nd}$$
 (8)

to experimental data (dashed curves). The near-field spectra of film A (red) and sapphire substrate (black) are well fitted. Most importantly, the partially screened sapphire phonon responses and the local resonators of the gold films can be well reproduced.

From the above analysis, extra information of the local resonators that is missing from the macroscopic EMA modeling can be readily extracted from the near-field investigation. These local resonators contribute to the far-field spectra significantly, similar to an array of antennas. It is therefore reasonable to reconstruct better modeling of the far-field reflectance from the obtained near-field information. That is, the contributions from the local resonators should be included in the EMA permittivity. The resonance frequency  $\omega_{0j}$ , the oscillator strength  $s_j$ , and the damping coefficient  $\gamma_j$  of the local resonators are expected to follow a specific distribution function  $D(\omega_0, s, \gamma)$ . That is, the macroscopic permittivity of the gold film is then given by

$$\varepsilon_{\rm eff}' = \varepsilon_{\rm eff} + \int_{\omega_{\rm min}}^{\omega_{\rm max}} D(\omega_0, s, \gamma) \frac{s}{\omega_0^2 - \omega^2 + i\gamma} d\omega_0 \qquad (9)$$





instead of simply  $\varepsilon_{\rm eff}$  given by Eq. (1). The exact form of  $D(\omega_0, s, \gamma)$  is complex and intimately related to the particular gold films under investigation. As a proof-of-concept demonstration, here we assume that  $D(\omega_0, s, \gamma)$  follows a constant distribution in the range from  $\omega_{\min} = 850 \text{ cm}^{-1}$  to  $\omega_{\text{max}} = 1600 \text{ cm}^{-1}$ . In this case,  $D = \frac{1}{\omega_{\text{max}} - \omega_{\text{min}}}$ . We take *s* and  $\gamma$  as constants for all resonators, but in principle they can be varied as well. Far-field reflectances calculated using  $\varepsilon'_{\rm eff}$  (red curve) and  $\varepsilon_{\rm eff}$  (blue curve) are shown in Fig. 4(f). With local resonators added to the picture (red curve) in this simplistic consideration, the spectral response is already significantly more consistent with the experimental data (dashed red curve). As can be observed in Figs. 4(a)-4(d), the resonators are generally weaker in IFs than in the CFs, which also explains the lower farfield reflectance in IF above  $\sim 900 \text{ cm}^{-1}$  [Fig. 1(b)]. A flowchart of our improved fitting procedure using a combination of the near-field and far-field data is summarized in Fig. 5.

#### 5. CONCLUSION AND SUMMARY

To conclude, we have investigated gold PMFs on sapphire substrates with far-field FTIR, IR, and THz nanoimaging as well as IR nano-spectroscopy. A wide range of frequencies from the mid-IR to the THz regime have been covered for the first time to our knowledge. Cluster-like inhomogeneous optical responses are clearly observed at mid-IR frequencies but are completely absent at much longer wavelengths in the THz regime. The elevated far-field reflectance above  $\sim 900 \text{ cm}^{-1}$  is found to be a direct consequence of the collective response of micro-resonators formed by the local gold clusters. Standard EMA methods do not address the local resonances and therefore underestimate the IR reflectance at certain frequency ranges. We conclude that a collaborative investigation of both far-field and near-field properties paints a more complete picture of the system under study: EMA provides a general background effective permittivity of the percolating gold films, which can be augmented by contributions from local resonators probed by near-field optics. This work can be instructional for future studies of random or quasi-random photonic structures using near-field methods, which are potentially important for applications of field enhancement, light trapping, and energy storage [44,45].

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