# Alignment of an off-axis parabolic mirror with two parallel $\mathrm{He}-\mathrm{Ne}$ laser beams 

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#### Abstract

Two parallel laser beams are used to align an off-axis parabolic mirror without alignment telescope and reference flat. In this method the optical axis of an off-axis parabolic mirror is made parallel to the incident laser beams, in the plane of incidence, by measuring directions of the reflected beams and by changing height and orientation of the mirror. Then, the focal point of the off-axis parabolic mirror is automatically found where the two reflected beams cross each other. The alignment of the optical axis to the incident beams is done without knowing focal length and off-axis distance of the mirror. Alignment sensitivity is derived both numerically and analytically. When focal length is 457 mm , off-axis distance is 127 mm , and diameter is 178 mm , the off-axis parabolic mirror is aligned to the incident beams with an angular error of less than 3 mrad.


Subject terms: off-axis parabolic mirrors; optical alignment; laser beams.
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## 1 Introduction

An optical system that employs reflective surfaces (spherical, paraboloidal, ellipsoidal, and so on) has many advantages over a refractive optical system. An all-reflective optical system has a wide bandwidth, which is determined by surface coating; at shorter wavelengths, surface accuracy and quality are critical specifications. Since there is no chromatic aberration, reflective optics may be assembled and tested at one wavelength without worrying about realigning at other wavelengths. ${ }^{1}$ This is a most convenient and powerful means of testing wideband reflective optical systems. Currently, axially symmetric mirrors are manufactured up to 160 in . in diameter and off-axis segments up to 80 in . in diameter with high tolerance. ${ }^{2}$ However, it is increasingly difficult to obtain high-quality refractive lenses at such large sizes. Also, in this case, doublet and triplet lenses become extremely heavy and difficult to mount. Reflective optical systems generally have fewer optical surfaces compared with refractive optical systems, which means fewer places for losses to occur and thereby higher system efficiency. ${ }^{3}$

The parabola is one of the best recognized aspheric surface geometries and is most frequently used in collimation or target simulation applications. When collimated light is incident parallel to the axis of revolution (or the optical axis), a concave parabolic surface focuses the light into a perfectly corrected point image on the optical axis. Conversely, a concave parabolic surface produces a highly collimated beam of light from a point source placed at its focal point.

[^0]In some reflective optical systems the focal point must be placed out of the incident collimated beam. ${ }^{4-6}$ The most frequently used optics, in this case, is an off-axis parabolic mirror that is a segment cut out of a large parabola. The optical axis of an off-axis parabolic mirror is the same as that of the parent parabola, which is the axis of revolution, and is, by definition, the physical center of the parent mirror. The position of the optical axis is specified by the distance measured from the inner edge of the mirror, which is called the off-axis distance.

Off-axis parabolic mirrors are manufactured in two different ways. In the first method, a small circular segment is cut from a large parabolic mirror. In this configuration the back surface of the off-axis parabolic mirror is approximately perpendicular to the side surface and to the optical axis. In the second method, an off-axis parabolic mirror is directly manufactured as an off-axis segment (stand-alone configuration). In this case, even if the back surface is perpendicular to the side surface, there is no relation between the back surface and the optical axis. ${ }^{7}$

Mirror manufacturers usually employ a sophisticated interferometer, such as laser unequal path interferometer, to test the optical surface accuracy of an off-axis parabolic mirror and measure mirror characteristics, such as focal length, off-axis distance, and optical axis. ${ }^{8}$ The optical axis is specified in two ways. First, mirror manufacturers put two pencil marks at opposite side surfaces of the mirror to show the intersection of a plane, which contains the optical axis, with the mirror. Second, mirror manufacturers attach a reference flat, mostly on expensive mirror mounts, perpendicular to the optical axis. However, to utilize the reference flat an expensive aligning tool, such as collimator or alignment telescope, is required. When there is no reference flat, one can locate the position of the focal point only by linear translations from the inner edge of the offaxis parabolic mirror, which are specified by manufacturers


Fig. 1 Top view of optical table for vertical alignment.
as off-axis distance and focal length. Since the translations are done in three-dimensional space, including the height measured from the surface of optical table, a large error is unavoidable in this case.

In this paper a new method for aligning an off-axis parabolic mirror without a reference flat is proposed. In this method an off-axis parabolic mirror is first adjusted so that its optical axis is in the plane made by two parallel incident beams. Next, the off-axis parabolic mirror is adjusted again so that the optical axis is parallel, in the plane of the incident beams, to the direction specified by the incident beams. This method makes it possible to align an off-axis parabolic mirror without knowing the exact focal length and off-axis distance. The equipment needed in this method includes a $\mathrm{He}-\mathrm{Ne}$ laser, a beamsplitter, an iris diaphragm, and plane mirrors.

## 2 Alignment Procedure

Figure 1 shows arrangement of the $\mathrm{He}-\mathrm{Ne}$ laser and offaxis parabolic mirror to be aligned. A beam from the $\mathrm{He}-$ Ne laser is split by the beamsplitter (BS) into two beams of equal intensity. The beams are made parallel to the surface of optical table. To check the parallelism, we use an iris diaphragm, which can be moved on an optical table with a fixed height, and we compare the beam spot with the iris opening at different locations on the table. The iris opening is made slightly smaller than the width of the beam spot so that the height difference may be easily seen. The two beams are also made parallel to each other. The parallelism, in this case, is checked by measuring the distance between the beams at different locations on the table, or by retro-reflecting the beams with a large plane mirror and observing how the retro-reflected beams overlap at a position between the laser and beamsplitter. The distance between the beams $\Delta x$ is determined by the diameter of the off-axis parabolic mirror. It is shown later that the optical axis can be aligned to the incident laser beams more accurately when $\Delta x$ is larger.


Fig. 2 Side view of optical table for vertical alignment.

In this paper a rectilinear coordinate system of an offaxis parabolic mirror is defined such that the $z$ axis is parallel to the optical axis and the $x$ axis is parallel to the surface of an optical table with its origin at the vertex of the parent parabola. The positive $y$ direction is from optical table to ceiling, as can be seen in Fig. 2. However, it is not necessarily perpendicular to the surface of the optical table, because the off-axis parabolic mirror is rotated about the $x$ axis during alignment. In the first alignment, which we call vertical alignment, we place the optical axis of an off-axis parabolic mirror in the plane made by two parallel incident beams. In the second alignment, which we call horizontal alignment, we place the optical axis parallel to the incident beams. The focal point of the off-axis parabolic mirror is found where the two reflected beams cross each other.

### 2.1 Vertical Alignment

The following steps are taken in the vertical alignment procedure:

1. The off-axis parabolic mirror is first put on the optical table such that its optical axis is nearly parallel to the incident beams. We call the incident beams beam 1 and beam 2, where beam 1 is farther away from the optical axis than beam 2 (see Fig. 1).
2. Then we make reflected beam 1 parallel to the surface of the optical table by rotating the mirror about the $x$ axis. To check parallelism we monitor the height of reflected beam 1 with the same iris that is used to make the incident beams parallel. Since incident laser beams have nonzero beamwidths, reflected beam 1 first converges to a small spot and then diverges. We put the iris at the position where reflected beam 1 becomes slightly larger than the opening of the iris and compare the beam spot with the iris opening.
3. Next we monitor height of reflected beam 2 with the same iris to see if the beam is parallel to the surface of the optical table. In this case the same method is used as for reflected beam 1.
4. If reflected beam 2 propagates upward, we decrease the height of the off-axis parabolic mirror.
5. Similarly, if reflected beam 2 propagates downward, we increase the height of the mirror (see Fig. 2).

The height is then adjusted and steps 2 to 5 are repeated until both reflected beams 1 and 2 are parallel to the surface
of the optical table. This will necessarily put the optical axis of the off-axis parabolic mirror in the plane made by the two incident beams and make the $y$ axis perpendicular to the surface of the optical table.

### 2.2 Horizontal Alignment

After vertical alignment, both reflected beams come to a focus at the same point in the plane of incidence. This point is not necessarily the focal point of the off-axis parabolic mirror because the incident beams may not be parallel to the optical axis. To perform horizontal alignment:

1. We put a plane mirror at the point where the reflected beams cross each other.
2. We then tilt the plane mirror to send one of the beams to a point at the off-axis parabolic mirror, which is between and above the two incident beams (see Figs. 3 and 4). This beam is called beam 3. We find the crossing point of the reflected beams by observing scattering of the beams from the surface of the plane mirror. By chopping one of the incident beams with a piece of paper we can easily see how well the beams overlap at the surface of the plane mirror.
3. Next, we check the height of reflected beam 3 to see if the beam is parallel to the surface of the optical table. This is done with another iris diaphragm, which can be moved on the table with a fixed height. The height is checked at two different locations on the table by changing the size of the iris opening because beam 3 diverges slightly after reflecting from the offaxis parabolic mirror
4. If reflected beam 3 propagates upward, we rotate the off-axis parabolic mirror about $y$ axis in the direction of smaller angle of incidence for beams 1 and 2.
5. Similarly, if the beam propagates downward, we rotate the mirror in the direction of larger angle of incidence.

Steps 1 to 5 are repeated until reflected beam 3 is parallel to the surface of the optical table. During iteration of the procedure the plane mirror is fine adjusted to send beam 3 to the same point at the off-axis parabolic mirror. This is to observe an angular change of reflected beam 3 in a consistent manner for a small rotation of the off-axis parabolic mirror. When the parabolic mirror is aligned such that reflected beam 3 is parallel to the surface of the optical table, the optical axis becomes parallel to the incident beams 1 and 2 , and the focal point is found at a point where reflected beams 1 and 2 cross each other.

## 3 Numerical Simulations and Analytic Approximations

### 3.1 Numerical Simulations

In this section exact rays are numerically traced to obtain the sensitivity of our alignment method and to estimate alignment error. For convenience in calculation it is assumed that the off-axis parabolic mirror is fixed in space and the two parallel incident beams are rotated with respect to the mirror by angles $\phi$ and $\theta$. Here $\phi$ represents an angle sub-


Fig. 3 Top view of optical table for horizontal alignment. A plane mirror is put at the point where reflected beams 1 and 2 cross each other.


Fig. 4 Side view of optical table for horizontal alignment. An angular deviation of reflected beam 3 from the plane of the incident beams is related to a rotation of the optical axis about the $y$ axis.
tended by the optical axis and projection of the incident beams on the $x-z$ plane (see Fig. 1). $\theta$ represents a rotation of the two incident beams about the $x$ axis of the mirror (see Fig. 2). The distance between the beams is assumed to be $\Delta x$. The surface of the optical table is assumed to be always parallel to the plane made by the two incident beams regardless of the beam rotations. Our sign convention is such that $\phi$ is positive in Fig. 1 and $\theta$ is negative in Fig. 2.

A parabola is defined in our simulations as
$z=\frac{1}{2 r}\left(x^{2}+y^{2}\right)$,
where $r$ is the radius of curvature, which corresponds to twice the focal length. When values of $\phi, \theta, \Delta x$, and the position of beam 1 at the off-axis parabolic mirror ( $x_{1}, y_{1}$, $z_{1}$ ) are given, beam 2 is represented by a line that is produced by two intersecting planes. The planes are obtained from Figs. 1 and 2 as
$y-y_{1}=\left(z-z_{1}\right) \tan \theta$,
$x-x_{1}-\Delta x / \cos \phi=-\left(z-z_{1}\right) \tan \phi$.
From Eqs. (1) to (3) the position of beam 2 at the parabolic mirror $\left(x_{2}, y_{2}, z_{2}\right)$ is obtained. Then, the surface normals
at the given beam positions are obtained as $\mathbf{N}_{1}=a_{1}\left(-x_{1}\right.$, $\left.-y_{1}, r\right)$ and $\mathbf{N}_{2}=a_{2}\left(-x_{2},-y_{2}, r\right)$, where $a_{1}$ and $a_{2}$ are normalization constants. Since incident beams 1 and 2 are parallel to each other, their unit wave vectors should be the same and are given by the rotation angles as $\mathbf{K}_{1}=\mathbf{K}_{2}=b(\tan \phi$, $-\tan \theta,-1$ ), where $b$ is normalization constant. Then, the unit wave vectors of the reflected beams 1 and 2 are obtained from Snell's law as
$\mathbf{R}_{1}=-2 \mathbf{N}_{1}\left(\mathbf{N}_{1} \cdot \mathbf{K}_{1}\right)+\mathbf{K}_{1}$,
$\mathbf{R}_{2}=-2 \mathbf{N}_{2}\left(\mathbf{N}_{2} \cdot \mathbf{K}_{2}\right)+\mathbf{K}_{2}$.
Since the off-axis parabolic mirror is rotated about the $x$ axis in the vertical alignment to make reflected beam 1 be in the plane made by the two incident beams, the rotation angle $\theta$ is given by
$\theta=\tan ^{-1}\left(N_{1 y} / N_{1 z}\right)$,
where $N_{1 y}$ and $N_{1 z}$ are $y$ and $z$ components of the unit surface normal $\mathbf{N}_{1}$ at the position of beam 1. Next, the surface normal to the plane made by the two incident beams is obtained as
$\mathbf{N}_{\mathrm{inc}}=c\left(0, N_{1 z},-N_{1 y}\right)$,
where $c$ is normalization constant. Then, the angular deviation of the reflected beam 2 is measured with respect to the surface of the optical table (or the plane made by the incident beams) and is derived as
$\Delta \theta=\pi / 2-\cos ^{-1}\left(\mathbf{R}_{2} \cdot \mathbf{N}_{\mathrm{inc}}\right)$.
Beam 3 is similarly traced in the horizontal alignment. First, the crossing point of reflected beams 1 and $2\left(x_{c}, y_{c}, z_{c}\right)$ is obtained from the incident beam positions at the off-axis parabolic mirror, $\left(x_{1}, y_{1}, z_{1}\right)$ and ( $x_{2}, y_{2}, z_{2}$ ), and the unit wave vectors of reflected beams 1 and 2 . It can be shown that $x$ and $z$ coordinates of the crossing point should satisfy the following equations in the $x-z$ plane of the mirror:
$x-x_{1}=\frac{R_{1 x}}{R_{1 z}}\left(z-z_{1}\right)$,
$x-x_{2}=\frac{R_{2 x}}{R_{2 z}}\left(z-z_{2}\right)$,
where the first subscript represents a beam and the second a component of a vector. In this case $y_{c}$ is assumed to be zero because the vertical alignment is already done. When the position of beam 3 is given by $\left(x_{3}, y_{3}, z_{3}\right)$ at the offaxis parabolic mirror, the surface normal at the beam position is obtained as $\mathbf{N}_{3}=d\left(-x_{3},-y_{3}, r\right)$ and the unit wave vector of beam 3 as $\mathbf{K}_{3}=e\left(x_{3}-x_{c}, y_{3}-y_{c}, z_{3}-z_{c}\right)$, where $d$ and $e$ are normalization constants. Next, the unit wave vector of reflected beam 3 is obtained from Snell's law as $\mathbf{R}_{3}=-2 \mathbf{N}_{3}\left(\mathbf{N}_{3} \cdot \mathbf{K}_{3}\right)+\mathbf{K}_{3}$. Then, the angular deviation of reflected beam 3 is derived from $\mathbf{R}_{3}$ as
$\left.\Delta \phi=\tan ^{-1}\left[R_{3 y} /\left(1-R_{3 y}\right)^{2}\right)^{1 / 2}\right]$
where $R_{3 y}$ is the $y$ component of the unit wave vector $\mathbf{R}_{3}$.

### 3.2 Analytic Approximations

In the vertical alignment $\theta, y_{1}$, and $y_{2}$ converge to zero as the optical axis of the off-axis parabolic mirror comes to be in the plane made by the two incident beams. In the horizontal alignment $\phi$ converges to zero as the optical axis becomes parallel to the incident beams. In this section we obtain approximate analytic solutions of $\Delta \theta$ and $\Delta \phi$ by assuming $y_{1}$ and $y_{2}$ to be much less than $x_{1}, x_{2}$, and $\Delta x$, and by assuming $\theta$ and $\phi$ to be much less than unity. Note that $\Delta x$ can be comparable to $x_{1}$ in magnitude. By using these assumptions we obtain an approximate expression for the angular deviation of reflected beam 2, in the vertical alignment, as
$\Delta \theta \approx 2 \Delta x x_{1} y_{1} / r^{3}$,
where $x_{1}$ and $y_{1}$ are $x$ and $y$ coordinates of beam 1 at the off-axis parabolic mirror, $\Delta x$ is the distance between the two incident beams, and $r$ is the radius of curvature. In the derivation the angle $\phi$ is assumed to be zero. Since beam 1 is farther away from the optical axis than beam $2, \Delta x$ always has the opposite sign of $x_{1}$ and values between zero and $-x_{1}$.

The smallest value of $\Delta \theta$ that can be detected in the laboratory depends totally on the angle measuring equipment. When the angular resolution is given, Eq. (11) shows that $y_{1}$ becomes smaller for larger values of $\left|\Delta x x_{1}\right|$ and/or for smaller radius of curvature. Here $y_{1}$ is the distance of beam 1 measured from $x-z$ plane at the parabolic mirror. Since the off-axis parabolic mirror is rotated about the $x$ axis for a nonzero value of $y_{1}$, the finite resolution in measuring $\Delta \theta$ causes a small rotation of the optical axis with respect to the plane of the incident beams (or the surface of optical table) after the vertical alignment is done. The angular error in this case is estimated as $y_{1} / r$, which is the opposite to the angle subtended by the surface normal at the position of beam 1 and the plane of the incident beams measured before the rotation of the mirror.

Predictions of Eq. (11) are compared with those of the exact numerical calculations. When $\left|x_{1}\right| \leq 0.5 f$ and $|\Delta x| \geq$ $0.1\left|x_{1}\right|$, the analytic predictions are good with an error less than $5 \%$ in the range $100 \mathrm{~mm} \leq f \leq 3000 \mathrm{~mm}$, where $f$ is the focal length of the off-axis parabolic mirror. Similarly, when $\left|x_{1}\right| \leq f$ and $|\Delta x| \geq 0.2\left|x_{1}\right|$, the analytic predictions are good with an error less than $15 \%$ in the range $100 \mathrm{~mm} \leq f$ $\leq 3000 \mathrm{~mm}$. In both cases the error itself is insensitive to $y_{1}$.

After the vertical alignment is done, reflected beams 1 and 2 cross each other in the plane of the incident beams. When $\phi$ is nonzero, the beam crossing point deviates from the focal point of the off-axis parabolic mirror in both the $x$ and $z$ directions. These small deviations cause reflected beam 3 to emerge out of the plane of the incident beams. Here an approximate analytic solution of $\Delta \phi$ is derived from the small deviations, $\Delta x_{c}$ and $\Delta z_{c}$. In this case the angle $\theta$ is assumed to be zero since the vertical alignment is already done:
$\Delta \phi=\frac{2 y_{3}\left(\Delta x_{c} x_{3}-\Delta z_{c} r\right)}{\left(x_{3}^{2}+y_{3}^{2}+r^{2}\right)\left(x_{3}^{2}+y_{3}^{2}+f^{2}\right)^{1 / 2}}$,

$$
\begin{equation*}
\Delta x_{c}=-\frac{2 \Delta z_{c} x_{1}}{r}+\phi f\left(1+\frac{3 x_{1}^{2}}{r^{2}}\right) \tag{13}
\end{equation*}
$$

$\Delta z_{c}=3 / 2 \phi\left(x_{1}+1 / 2 \Delta x\right)$,
where $\left(x_{3}, y_{3}, z_{3}\right)$ is the position of beam 3 at the off-axis parabolic mirror. It is noted from the equations that $|\Delta \phi|$ becomes larger for larger values of $\left|x_{1}\right|$ and $\left|y_{3}\right|$, and for smaller values of $|\Delta x|$ and $\left|x_{3}\right| . \Delta x$ is assumed to be between 0 and $-x_{1}$ so that both $x_{1}$ and $x_{2}$ have the same sign. When $\Delta x$ is larger than $-x_{1}$, the angular deviation $\Delta \phi$ is the same as is obtained for $\Delta x=2 x_{1}-\Delta x$. It is also noted that the angular error of the focal point, $\phi$, is related to $\Delta \phi$ through $\Delta x_{c}$ and $\Delta z_{c}$.

Predictions of Eq. (12) are compared with those of the exact numerical calculations. In this case $x_{3}$ is assumed to be $\left(x_{1}+x_{2}\right) / 2$. When $0.1 f \leq\left|x_{1}\right| \leq f$ and $25 \mathrm{~mm} \leq|\Delta x| \leq\left|x_{1}\right|$, the analytic predictions are good with an error less than $17 \%$. The error sharply increases when $\left|x_{1}\right|$ becomes smaller than $0.1 f$. For example, when $\left|x_{1}\right|=0.05 f$ and $25 \mathrm{~mm} \leq$ $|\Delta x| \leq\left|x_{1}\right|$, the analytic predictions are good with an error less than $29 \%$.

The alignment errors $\theta$ and $\phi$ can be converted to other system performance criteria such as rms wave-front error, P-V wave-front error, or Strehl ratio. For an off-axis parabolic mirror with a focal length of 457 mm , an off-axis distance of 127 mm , and a diameter of 178 mm , an alignment error in the vertical direction, $\theta \leq 5.0 \mathrm{mrad}$, is linearly related to the rms wave-front error, rms WFE $\leq 7.1 \mu \mathrm{~m}$. Similarly, an alignment error in horizontal direction, $\phi \leq$ 5.0 mrad , is linearly related to the rms wave-front error, rms WFE $\leq 4.0 \mu \mathrm{~m} .{ }^{9}$

## 4 Experiment and Discussions

Our two-laser-beam method is used to align an off-axis parabolic mirror whose focal length is 457 mm , off-axis distance is 127 mm , and diameter is 178 mm . First, a 633nm beam from an $\mathrm{He}-\mathrm{Ne}$ laser is split by a beamsplitter into two beams of equal intensity. The beam diameter is measured as $3.0 \pm 0.3 \mathrm{~mm}$ at the off-axis parabolic mirror, and the beam divergence angle is measured as $1.5 \pm 0.1 \mathrm{mrad}$. The beams are made parallel to each other with an error of $\pm 0.3 \mathrm{mrad}$. The beams are also made parallel to the surface of the optical table with an error of $\pm 0.1 \mathrm{mrad}$ (the flatness of our optical table is $\pm 0.005$ in. over $5 \times 10$ in. ${ }^{2}$ ). The parallelism of the beams is measured with an iris diaphragm whose opening is adjusted to be slightly smaller than the width of the beam. The distance between the two incident beams is measured as 90 mm . The distance of beam 1 is 185 mm from the optical axis at the off-axis parabolic mirror.

In the vertical alignment the off-axis parabolic mirror is first raised such that the optical axis is above the plane of the incident beams. Then, the height of the mirror is decreased step by step, with a decrement of $\sim 1 \mathrm{~mm}$, until reflected beams 1 and 2 become parallel to the plane of the incident beams. After the alignment the height of the beams is marked at the edge of the mirror. Next, the off-axis parabolic mirror is lowered such that the optical axis is below the plane of the incident beams. Then, the height of
the mirror is increased step by step, with an increment of $\sim 1 \mathrm{~mm}$, until reflected beams 1 and 2 become parallel to the plane of the incident beams. Similarly, the height of the beams is marked at the same edge of the mirror. It is noted that the first mark is below the second and that the mark of the exact optical axis, which is provided by the mirror manufacturer, is in between the two marks. The distance between the first and the manufacturer's mark is almost the same as that between the second and the manufacturer's mark, and is measured as $1.5 \pm 0.5 \mathrm{~mm}$. The height order among the three marks is always the same for several repetitions of the above procedure. The alignment error of $1.5 \pm 0.5 \mathrm{~mm}$ can be explained as follows. The smallest angular change we can detect in our laboratory, using an iris diaphragm, is estimated as $\sim 0.1 \mathrm{mrad}$. The exact numerical and analytic solutions of $\Delta \theta$ predict that reflected beam 2 deviates from the plane of the incident beams by $\pm 0.1 \mathrm{mrad}$ when the position of beam 1 is $\sim 3 \mathrm{~mm}$ below (or above) the plane of the incident beams in our experimental conditions. Since we are able to detect the beam deviation of $\sim \pm 0.1 \mathrm{mrad}$, we automatically decrease (or increase) the height of the mirror by the step of $\sim 1 \mathrm{~mm}$. Then, the angular deviation of reflected beam 2 becomes smaller than the angular resolution in our laboratory, which terminates the alignment procedure with an error of $y_{1} \approx$ 2 mm .

In the horizontal alignment the plane mirror is tilted to make beam 3 land at the point $x_{3}=-140 \mathrm{~mm}$ and $y_{3}=64$ mm in the off-axis parabolic mirror. The parabolic mirror is rotated about $y$ axis by a step of $\sim 1.5 \mathrm{mrad}$ until reflected beam 3 is parallel to the surface of the optical table. Then, the crossing point of reflected beams 1 and 2 is marked by a sharp pointer (a needle on mount). The off-axis parabolic mirror is rotated further in the same direction, by a large amount this time, and the previous alignment procedure is repeated until reflected beam 3 again becomes parallel to the surface of optical table. The new beam crossing point is marked by another pointer. As in the vertical alignment the two marks do not cross each other at several repetitions of the procedure. The angular difference between the marks is measured as $3.0 \pm 0.5 \mathrm{mrad}$ in the plane of the incident beams. The numerical and analytic solutions of $\Delta \phi$ predict that reflected beam 3 deviates from the plane of the incident beams by $\sim 0.1 \mathrm{mrad}$ when an angular difference between the crossing point and the focal point is $\sim 3 \mathrm{mrad}$ in our experimental conditions, which is consistent with the observation.

To check the accuracy of the two-laser-beam method we use an alignment telescope and the reference flat that is attached to our off-axis parabolic mirror. This method shows that the focal point found in the two-laser-beam method is off from the exact focal point by $\pm 2 \mathrm{mrad}$ in vertical direction and by $\pm 3 \mathrm{mrad}$ in horizontal direction. The vertical error is consistent with that predicted by the numerical and analytic solutions of $\Delta \theta$. Equation (11) shows that the minimum angular resolution of $\sim 0.1 \mathrm{mrad}$ in $\Delta \theta$ corresponds to $\sim 2 \mathrm{~mm}$ of $y_{1}$, which is then converted to an error of $\sim 2 \mathrm{mrad}$ in $\theta$. Similarly, we note that the horizontal error is consistent with that predicted by the numerical and analytic solutions of $\Delta \phi$. Equation (12) shows that the minimum resolution of $\sim 0.1 \mathrm{mrad}$ in $\Delta \phi$ corresponds to $\sim 3 \mathrm{mrad}$ in $\phi$ in our experimental conditions.

## 5 Conclusions

Two parallel laser beams are used to align an off-axis parabolic mirror without alignment telescope and reference flat. The alignment sensitivity of two-laser-beam method is derived both numerically and analytically. When the minimum angular resolution of beam direction is given, the alignment error can be estimated from the numerical and analytic solutions.

Since two parallel laser beams can be easily folded by a large plane mirror, the two-laser-beam method can be used to make optical axes of two different off-axis parabolic mirrors parallel to each other or to match the focal points of two different off-axis parabolic mirrors. In these cases it is not necessary to know the focal lengths and off-axis distances of the parabolic mirrors.

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